5.6 Notes and Examples

Name: Block: Seat:

L'Hôpital's Rule

1. Warm Up 1: Remember Limits? First substitute to determine if the limit is Type I, II, or III.
   (a) $\lim_{x\to 1} \dfrac{x^3-1}{x^2+x+1}$

   (b) $\lim_{x\to 2} \dfrac{2}{x^2-4}$

   (c) $\lim_{x\to 2} \dfrac{x^2+x-6}{x^2-4}$

2. Warm Up 2: Try these "Type III" limits with a calculator (Recall the Graph and Table Methods?)
   (a) $\lim_{x\to 0} \left(\dfrac{e^x-1}{x}\right)$

   (b) $\lim_{x\to 1^+} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right)$

   (c) $\lim_{x\to \infty} \left(1+\dfrac{2}{x}\right)^x$

3. Warm Up 2: Try these "Type III" limits by either factoring or multiplying top and bottom with the conjugate:
   (a) $\lim_{x\to -1} \left(\dfrac{2x^2-2}{x+1}\right)$

   (b) $\lim_{x\to \infty} \left(\dfrac{3x^2-1}{2x^2+1}\right)$

   (c) $\lim_{x\to 7} \left(\dfrac{\sqrt{x+2}-3}{x-7}\right)$

Background of the new method for "Type III" limits: L'Hôpital's Rule

1. Named after Guillaume de L'Hôpital, who published in the first ever differential calculus textbook
2. Actually invented/discovered by Swiss mathematician Johann Bernoulli
3. The method uses derivatives to evaluate indeterminate limits.
4. Don't try it on $\lim_{x \to 3} \left(\frac{2x+7}{4x+1} \right)$. Can you guess why?

Now the new method: L'Hôpital's Rule

1. Sometimes, you are not able to simplify equations after doing direct ______________
2. When this happens, it is called an ___________________ form.
3. If $\lim_{x \to c} \frac{f(x)}{g(x)}$ is of the form ______________ or ______________, then $\lim_{x \to c} \frac{f(x)}{g(x)} =$
4. In order to use L'Hôpital's Rule you must
   (a) Write the expression in _________________ form
   (b) State the it is either ______________ form or ______________ form, and you are using ______________ rule. (most abbreviations are accepted)
   (c) Take the limit of numerator's derivative _________________ the denominator's derivative.

4. Examples
   (a) $\lim_{x \to \infty} \frac{x}{e^x}$

   (b) $\lim_{x \to \infty} \frac{e^x}{x}$

   (c) $\lim_{x \to 0} \frac{x}{e^x}$

(d) $\lim_{x \to -\infty} x^2 e^x$
Hint: you need to write this as a fraction

(e) $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$

(f) $\lim_{x \to 0} \frac{4e^{2x} - 4}{x}$

5. There are other indeterminate forms that you can use L'Hôpital's Rule with, but you first need to make the expression into a ratio (fractional form)

    1. _______________
    2. _______________
    3. _______________
    4. _______________
    5. _______________

    (a) 1^∞ form: $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^x$

    (b) ∞^0 form: $\lim\limits_{x\to\infty} x^{1/x}$

    (c) 0^0 form: $\lim\limits_{x\to 0^+} x^x$

(d) $0 \cdot \infty$ form: $\lim\limits_{x \to \infty} e^{-x}\sqrt{x}$

(e) $\infty - \infty$ form: $\lim\limits_{x \to 1^+} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1} \right)$

Bonus round (not technically L'Hôpital, but using the same idea about rates):

Using Relative Growth Rates to Evaluate a Limit to $\pm\infty$

When evaluating limits of the the form $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)}$, then the limit is

1. $\pm\infty$ if $f(x)$ grows ________________ than $g(x)$.

2. 0 if $f(x)$ grows ________________ than $g(x)$.

As $x \to \infty$

$$x^x \succ x! \succ a^x \succ x^a \succ \log_a x$$

6. Examples

   (a) $\lim\limits_{x\to\infty} \dfrac{e^x}{4^x - 1}$

   (b) $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^3 + 4}$

   (c) $\lim\limits_{x\to\infty} \dfrac{x^3 - 2x + 1}{3x^4 + 3x - 7}$

   (d) $\lim\limits_{x\to\infty} \dfrac{x^x}{x!}$